Thermodynamics of space quanta models quantum gravity

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Abstract

Canonically quantized $3+1$ general relativity with the global one dimensionality (1D) conjecture defines the model, which dimensionally reduced and secondary quantized yields the 1D quantum field theory wherein generic one-point correlations create physical scales.

This simple quantum gravity model, however, can be developed in a wider sense. In this paper we propose to consider *ab initio* thermodynamics of space quanta as the quantum gravity phenomenology. The thermodynamics is constructed in the entropic formalism.

Keywords quantum gravity models ; $3+1$ general relativity ; low dimensional quantum field theories ; global one-dimensionality ; thermodynamics of space quanta.

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1 Introduction

Both the theory and phenomenology of quantum gravity possess the most fundamental meaning for the contemporary theoretical physics. Possibly the theory of quantized gravitational fields will able to predict unknown facts and open way for new physics. The efforts of many generations of physicists and mathematicians working on quantum gravity unquestionably have given significant contribution to science. In this a great success, however, understanding the physical role of quantum gravity seems to be still very distant and intriguing perspective (For some proposals see *e.g.* Ref. [1]).

In this paper we discuss the next implication following form the simple model of quantum gravity [2] having strict roots in the generic quantum cosmology [3]. The model was constructed within the Wheeler–DeWitt theory, called quantum geometrodynamics, with taking into account the global one-dimensional conjecture. The conjecture states that geometrodynamical wave functions are dependent on the one dimension only, that is an embedding volume form. It follows from the assumption that matter fields are functional of a volume form only. It reduces the Wheeler–DeWitt theory to the superspace strata, called minisuperspace. By application of the dimensional reduction the resulting model can be presented in the Dirac equation form with the Euclidean Clifford algebra $\mathcal{C}\ell_{1,1}(\mathbb{R})$, and by appropriate diagonalization procedure the equation can be quantized in the Fock space. Obtained 1D quantum field theory defines quantum gravity model, wherein quantum correlations yield physical scales.

However, the investigated simple model of quantum gravity can be developed and applied. This paper gives one of its possible physical implications, that is thermodynamics of space quanta. By space quanta we understand quantum states of a 3-dimensional embedding. The Fock space formulation gives a possibility to consider density matrix related to the model, and build formal thermodynamics. As the example we are discussing the one-particle approximation. Entropy and energy are calculated, and their appropriate renormalization is done. In result we obtain the 2nd order Eulerian system, and all thermodynamic quantities are calculated in frames of the standard statistical mechanics by application of first principles only, *i.e.* thermodynamics is done *ab initio*. In this way we receive the model of quantum gravity strictly related to phenomenology.

Structurally the paper is organized as follows. The preliminary section 2 briefly presents the simple model of quantum gravity. Section 3 is devoted to the development of the model, that is the thermodynamics of space quanta. In the final section 4 the new results are discussed.

2 The simple quantum gravity model

Let us summarize the quantum gravity model [2]. Regarding general relativity [4] spacetime is a 4-dimensional pseudo-Riemannian manifold (M, g) with a metric $g_{\mu\nu}$ satisfying the Einstein field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} {}^{(4)}R + g_{\mu\nu}\Lambda = 3T_{\mu\nu}, \tag{1}$$

where units $c = 8\pi G/3 = 1$ were used, $R_{\mu\nu}$ is the second fundamental form, ${}^{(4)}R$ is the scalar curvature, Λ is cosmological constant, and $T_{\mu\nu}$ is the stress-energy tensor arising from a Matter fields Lagrangian $\mathcal{L}_\psi$ by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S_\phi}{\delta g^{\mu\nu}} \quad , \quad S_\phi = \int d^4x \sqrt{-g} \mathcal{L}_\psi. \tag{2}$$

For M closed, having a boundary $(\partial M, h)$ with an induced metric h_{ij} and the Gauss curvature tensor K_{ij}, (1) are equations of motion for the Einstein–Hilbert action supplemented by the York–Gibbons–Hawking term [5]

$$S[g] = \int_M d^4x \sqrt{-g} \left\{ -\frac{1}{6} {}^{(4)}R + \frac{\Lambda}{3} \right\} + S_\phi - \frac{1}{3} \int_{\partial M} d^3x \sqrt{h} K, \tag{3}$$

where $K = h^{ij} K_{ij}$. One can parameterize a metric by the $3+1$ splitting [7]

$$g_{\mu\nu} = \left[\begin{array}{cc} -N^2 + N_i N^i & N_j \\ N_i & h_{ij} \end{array} \right] \quad , \quad N^i = h^{ij} N_j \quad , \quad h_{ik} h^{kj} = \delta_i^j, \tag{4}$$

which for stationary ϕ arises by a timelike Killing vector field and global spacelike foliation $t = const$ on M, and satisfies the Nash embedding theorem [6]. With this (3) takes the Hamilton form $S = \int dt L$ with the Lagrangian

$$L = \int_{\partial M} d^3x \left\{ \pi \dot{N} + \pi^i \dot{N}_i + \pi^{ij} \dot{h}_{ij} + \pi_\phi \dot{\phi} - NH - N_i H^i \right\}, \tag{5}$$

$$\pi_\phi = \frac{\partial L_\phi}{\partial \dot{\phi}} \quad , \quad \pi = \frac{\partial L}{\partial \dot{N}} \quad , \quad \pi^i = \frac{\partial L}{\partial \dot{N}_i}, \tag{6}$$

$$\pi^{ij} = \frac{\partial L}{\partial \dot{h}_{ij}} = \sqrt{h}\left(h^{ij} K - K^{ij} \right) \quad , \quad \dot{h}_{ij} = N_{i|j} + N_{j|i} - 2N K_{ij}, \tag{7}$$

$$H = \sqrt{h} \left\{ K^2 - K_{ij} K^{ij} + {}^{(3)}R - 2\Lambda - 6\varrho \right\} \quad , \quad H^i = -2\pi^{ij}{}_{;j}, \tag{8}$$

where $\varrho = n^\mu n^\nu T_{\mu\nu}$, $n^\mu = (1/N)\left[1, -N^i\right]$, ${}^{(3)}R = h^{ij} R_{ij}$. Time-preservation of the primary constraints [8, 9] yields the secondary ones

$$\pi \approx 0 \quad , \quad \pi^i \approx 0 \longrightarrow H \approx 0 \quad , \quad H^i \approx 0 \tag{9}$$

called the Hamiltonian (scalar) and the diffeomorphism (vector) constraint. Vector constraint reflects spatial diffeoinvariance, scalar one is dynamical. Regarding DeWitt [9] H^i generates the diffeomorphisms $\widetilde{x}^i = x^i + \xi^i$

$$i\left[h_{ij}, \int_{\partial M} H_a \xi^a d^3x\right] = -h_{ij,k}\xi^k - h_{kj}\xi^k{}_{,i} - h_{ik}\xi^k{}_{,j} \quad , \tag{10}$$

$$i\left[\pi^{ij}, \int_{\partial M} H_a \xi^a d^3x\right] = -\left(\pi^{ij}\xi^k\right)_{,k} + \pi^{kj}\xi^i{}_{,k} + \pi^{ik}\xi^j{}_{,k} \quad , \tag{11}$$

and the first-class constraints algebra holds

$$i\left[H_i(x), H_j(y)\right] = \int_{\partial M} H_a c^a_{ij} d^3z \quad , \quad i\left[H(x), H_i(y)\right] = H\delta^{(3)}_{,i}(x,y), \tag{12}$$

$$i\left[\int_{\partial M} H\xi_1 d^3x, \int_{\partial M} H\xi_2 d^3x\right] = \int_{\partial M} H^a\left(\xi_{1,a}\xi_2 - \xi_1\xi_{2,a}\right) d^3x. \tag{13}$$

Here $H_i = h_{ij}H^j$, and $c^a_{ij} = \delta^a_i \delta^b_j \delta^{(3)}_{,b}(x,z)\delta^{(3)}(y,z) - (i \leftrightarrow j, x \leftrightarrow y)$ are the structure constants of the spatial diffeomorphism group. The canonical quantization [8, 10]

$$i\left[\pi^{ij}(x), h_{kl}(y)\right] = \frac{1}{2}\left(\delta^i_k\delta^j_l + \delta^i_l\delta^j_k\right)\delta^{(3)}(x,y), \tag{14}$$

$$i\left[\pi^i(x), N_j(y)\right] = \delta^i_j \delta^{(3)}(x,y) \quad , \quad i\left[\pi(x), N(y)\right] = \delta^{(3)}(x,y), \tag{15}$$

applied to the Hamiltonian constraint into the Hamilton–Jacobi form [11]

$$G_{ijkl}\pi^{ij}\pi^{kl} - \sqrt{h}\left({}^{(3)}R - 2\Lambda - 6\varrho\right) = 0, \tag{16}$$

where G_{ijkl} is the Wheeler metric on superspace [12]

$$G_{ijkl} \equiv \frac{1}{2\sqrt{h}}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right), \tag{17}$$

yields the Wheeler–DeWitt equation [9, 13, 14] modeling quantum gravity

$$\left\{G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} + h^{1/2}\left({}^{(3)}R - 2\Lambda - 6\varrho\right)\right\}\Psi[h_{ij}, \phi] = 0. \tag{18}$$

Other first-class reflect diffeoinvariance of a wave function $\Psi[h_{ij}, \phi]$

$$\pi\Psi[h_{ij}, \phi] = 0, \quad \pi^i\Psi[h_{ij}, \phi] = 0, \quad H^i\Psi[h_{ij}, \phi] = 0. \tag{19}$$

The Wheeler–DeWitt equation (18) is independent on time quantum mechanics on the superspace of 3-dimensional embeddings. The simple model reduces the superspace to its strata, called minisuperspace.

The simple model assumes that Matter fields are one-dimensional (1D) functionals

$$\phi=\phi[h] \quad , \quad h=\frac{1}{3}\varepsilon^{ijk}\varepsilon^{abc}h_{ia}h_{jb}h_{kc} \quad , \tag{20}$$

where ε is the Levi-Civita tensor, so that the conjectured 1D wave functions

$$\Psi[h_{ij},\phi]\rightarrow\Psi[h], \tag{21}$$

satisfy the global one-dimensional evolution

$$\left\{G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}}+h^{1/2}\left({}^{(3)}R-2\Lambda-6\varrho[h]\right)\right\}\Psi[h]=0. \tag{22}$$

Assumption (21) is analogous to the generic model [3], but the 1D theory (22) holds for nonhomogeneous isotropic quantum cosmologies.

Considering the Jacobi rule for differentiation of a determinant [4] together with the $3+1$ splitting (4) one obtains

$$\delta g=gg^{\mu\nu}\delta g_{\mu\nu}\longrightarrow N^2\delta h=N^2hh^{ij}\delta h_{ij}, \tag{23}$$

which reduces the differential operator in (22)

$$\frac{\delta}{\delta h_{ij}}\Psi[h]=hh^{ij}\frac{\delta}{\delta h}\Psi[h]\longrightarrow G_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}}\Psi[h]=-\frac{3}{2}h^{3/2}\frac{\delta^2}{\delta h^2}\Psi[h], \tag{24}$$

and yields the simple 1D quantum gravity model

$$\left(\frac{\delta^2}{\delta h^2}-m^2\right)\Psi=0 \quad , \quad m^2=\frac{2}{3h}\left({}^{(3)}R-2\Lambda-6\varrho[h]\right). \tag{25}$$

where m is the mass of the classical field $\Psi[h]$. In fact (25) is a field-theoretic equation of motion $\delta S[\Psi]/\delta\Psi=0$ for the Euclidean action

$$S[\Psi]=\int\delta hL[\Psi,\Pi_\Psi] \quad , \quad L=\frac{1}{2}\Pi_\Psi^2+\frac{m^2}{2}\Psi^2, \tag{26}$$

where $\Pi_\Psi=\dfrac{\delta\Psi}{\delta h}$ is conjugate momentum which allows rewrite (25) in two-component model in the Dirac equation form

$$\left(i\gamma\frac{\delta}{\delta h}-M\right)\Phi=0 \quad , \quad \Phi=\left[\begin{array}{c}\Pi_\Psi\\ \Psi\end{array}\right] \quad , \quad M=\left[\begin{array}{cc}1 & 0\\ 0 & -m^2\end{array}\right], \tag{27}$$

with the Euclidean Clifford algebra $\mathcal{C}\ell_{1,1}(\mathbb{R})$ [15]

$$\gamma=\left[\begin{array}{cc}0 & -i\\ i & 0\end{array}\right] \quad , \quad \gamma^2=I \quad , \quad \{\gamma,\gamma\}=2\delta_E \quad , \quad \delta_E=\left[\begin{array}{cc}1 & 0\\ 0 & 1\end{array}\right], \tag{28}$$

having a $2D$ complex representation. Restricting to $Pin_{1,1}(\mathbb{R})$ yield a 2D spin representations; restricting to $Spin_{1,1}(\mathbb{R})$ splits it onto a sum of two 1D Weyl representations; $\mathcal{C}\ell_{1,1}(\mathbb{R})$ decomposes into a direct sum of two isomorphic central simple algebras or a tensor product

$$\mathcal{C}\ell_{1,1}(\mathbb{R}) = \mathcal{C}\ell^{+}_{1,1}(\mathbb{R}) \oplus \mathcal{C}\ell^{-}_{1,1}(\mathbb{R}) = \mathcal{C}\ell_{2,0}(\mathbb{R}) \otimes \mathcal{C}\ell_{0,0}(\mathbb{R}), \tag{29}$$

$$\mathcal{C}\ell_{1,1}(\mathbb{R}) \cong \mathbb{R}(2) \quad , \quad \mathcal{C}\ell^{\pm}_{1,1}(\mathbb{R}) = \frac{1 \pm \gamma}{2}\mathcal{C}\ell_{1,1}(\mathbb{R}) \cong \mathbb{R} \quad , \quad \mathcal{C}\ell_{0,0}(\mathbb{R}) \cong \mathbb{R}. \tag{30}$$

The Dirac equation (27) can be rewritten in the dynamical Fock reper $\mathfrak{B}$

$$\mathbf{\Phi} = \mathbb{Q}\mathfrak{B}, \quad \mathbb{Q} = \left[\begin{array}{cc} 1/\sqrt{2|m|} & 1/\sqrt{2|m|} \\ -i\sqrt{|m|/2} & i\sqrt{|m|/2} \end{array}\right], \tag{31}$$

$$\mathfrak{B} = \left\{ \left[\begin{array}{c} \mathsf{G}[h] \\ \mathsf{G}^{\dagger}[h] \end{array}\right] : \left[\mathsf{G}[h'], \mathsf{G}^{\dagger}[h]\right] = \delta\left(h' - h\right), \left[\mathsf{G}[h'], \mathsf{G}[h]\right] = 0 \right\}. \tag{32}$$

Determining a reper $\mathfrak{F}$ by the diagonalization due to the Bogoliubov transformation and the Heisenberg equations of motion

$$\mathfrak{F} = \left[\begin{array}{cc} u & v \\ v^{*} & u^{*} \end{array}\right]\mathfrak{B} \quad , \quad \frac{\delta\mathfrak{F}}{\delta h} = \left[\begin{array}{cc} -i\Omega & 0 \\ 0 & i\Omega \end{array}\right]\mathfrak{F}, \tag{33}$$

where $|u|^2 - |v|^2 = 1$, u, v, Ω are functionals of h, one obtains

$$\frac{\delta \mathbf{b}}{\delta h} = \mathbb{X}\mathbf{b} \quad , \quad \mathbf{b} = \left[\begin{array}{c} u \\ v \end{array}\right] \quad , \quad \Omega \equiv 0, \tag{34}$$

so that $\mathfrak{F}$ is the Fock the initial data static reper (I) with correct vacuum

$$\mathfrak{F} = \left\{ \left[\begin{array}{c} \mathsf{G}_I \\ \mathsf{G}^{\dagger}_I \end{array}\right] : \left[\mathsf{G}_I, \mathsf{G}^{\dagger}_I\right] = 1, \left[\mathsf{G}_I, \mathsf{G}_I\right] = 0 \right\} \quad , \quad \mathsf{G}_I \left|\mathrm{VAC}\right\rangle = 0, \tag{35}$$

and integrability of (34) can be done in the superfluid parametrization

$$u = \frac{\mu + 1}{2\sqrt{\mu}}e^{i\theta} \quad , \quad v = \frac{\mu - 1}{2\sqrt{\mu}}e^{-i\theta} \quad , \quad \theta = m_I \int_{h_I}^{h} \mu' \delta h', \tag{36}$$

where $\mu \equiv \mu[h]$, $\mu' = \mu[h']$ is a mass scale. In result one obtains the solution

$$\mathbf{\Phi} = \mathbb{Q}\mathbb{G}\mathfrak{F} \quad , \quad \mathbb{G} = \left[\begin{array}{cc} u^{\star} & -v^{\star} \\ -v & u \end{array}\right], \tag{37}$$

and particulary one establishes the field operator and the generic one-point correlator

$$\mathbf{\Psi} = \frac{1}{\sqrt{2m_I}}\left(\frac{e^{-i\theta}}{2\mu}\mathsf{G}_I + \frac{e^{i\theta}}{2\mu}\mathsf{G}^{\dagger}_I\right) \quad , \quad \left\langle \mathrm{VAC}\right| \mathbf{\Psi}^{\dagger}[h]\mathbf{\Psi}[h]\left|\mathrm{VAC}\right\rangle = \frac{1}{\mu^2}, \tag{38}$$

where the quantum correlator was normalized to unity in h_I. The static reper formulation defines the concept of space quanta - the quantized fields associated with an 3-dimensional embedding.

3 The thermodynamics

Thermodynamic equilibrium corresponding to quantum field theory in the static Fock reper, allows using of first principles of statistical mechanics [16], and formulate *ab initio* thermodynamics of space quanta. Let us test the one-particle density matrix approximation.

3.1 One-particle density matrix. Entropy and energy

In the one-particle approximation the density operator D is equivalent to an occupation number operator. Thermodynamic equilibrium is determined with respect to the static reper, so that the one-particle density matrix in equilibrium $\mathbb{D}$ is given by the Von Neumann–Heisenberg picture

$$\begin{aligned} \mathsf{D} &= \mathsf{G}^{\dagger}\mathsf{G} = \mathfrak{F}^{\dagger}\mathbb{D}\mathfrak{F}, \qquad (39)\\ \mathbb{D} &= \frac{1}{4\mu}\left[\begin{array}{cc} (\mu+1)^2 & 1-\mu^2 \\ 1-\mu^2 & (\mu-1)^2 \end{array}\right]. \qquad (40) \end{aligned}$$

Note that $\det\mathbb{D} = 0$, that means in the one-particle approximation the corresponding thermodynamics is not invertible. Employing (40) one can establish the occupation number value

$$N = \frac{\mathrm{Tr}\left(\mathbb{D}^2\right)}{\mathrm{Tr}\mathbb{D}} = \frac{\mu^2+1}{2\mu}, \qquad (41)$$

and the entropy can be derived from its basic definition

$$S = -\frac{\mathrm{Tr}(\mathbb{D}\ln\mathbb{D})}{\mathrm{Tr}\mathbb{D}} = \sum_{n=1}^{\infty}\sum_{k=1}^{n}\frac{(-1)^k}{n}\binom{n}{k-1}S_k, \qquad (42)$$

where $\binom{n}{m}$ are the Newton binomial symbols, and

$$S_k = \frac{\mathrm{Tr}(\mathbb{D}^k)}{\mathrm{Tr}\mathbb{D}} = N^{k-1}, \qquad (43)$$

are cluster entropies. The series (42) converges for the spectral radius values

$$\rho(\mathbb{D}-\mathbb{I}) < 1 \Longrightarrow \mu \in (1; 2+\sqrt{3}), \qquad (44)$$

or equivalently for $m \in (1; 2+\sqrt{3})m_I$, with the result

$$S = -\frac{\zeta(1)}{2}\left(\frac{\mu^2-1}{\mu^2+1}\right)^2 - \frac{\mu^4+6\mu^2+1}{(\mu^2+1)^2}\ln\frac{(\mu-1)^2}{2\mu}, \qquad (45)$$

where $\zeta(s) = \sum_{n=1}^{\infty} \dfrac{1}{n^s}$ is the Riemann zeta function; $\zeta(1)$ is formally infinite.

Note that by straightforward application of the Hagedorn hadronization formula $m \sim T_H$ [17], where m is the mass of the system, one can establishes the hadronized temperature as

$$\frac{T_H}{T_I} = \mu, \tag{46}$$

where $k_B T_I = m_I c^2$. By the relation $\langle m \rangle c^2 \sim k_B \langle T_H \rangle$ one obtains averaged hadronized temperature normalized to T_I value

$$\left\langle \frac{T_H}{T_I} \right\rangle = \langle \mu \rangle = \frac{1+\sqrt{3}}{2} \approx 2.732, \tag{47}$$

so that one can establish the ratio

$$\frac{\langle T_H \rangle}{T_H} \in \left(\frac{\sqrt{3}-1}{2}, \frac{\sqrt{3}+1}{2} \right) \approx (0.366, 1.366)\,. \tag{48}$$

Defining anisotropy as $\Delta T_H = \langle T_H \rangle - T_H$ one derives

$$\frac{\Delta T_H}{T_H} \in \left(\frac{\sqrt{3}-3}{2}, \frac{\sqrt{3}-1}{2} \right) \approx (-0.633; 0.366), \tag{49}$$

so that averaged anisotropy is

$$\left\langle \frac{\Delta T_H}{T_H} \right\rangle = 0.5. \tag{50}$$

By the first approximation, (47) can be identified with background temperature, *e.g.* for $T_I \sim 1K$ it is exactly averaged CMB radiation temperature. Next approximations of the density matrix or fuzzing of the interval $\mu \in (1; 2+\sqrt{3})$ will give next orders of the numbers.

In the static reper Hamiltonian matrix $\mathbb{H}$ of the system equals

$$\mathsf{H} = \frac{m}{2}\left(\mathsf{G}^\dagger \mathsf{G} + \mathsf{G}\mathsf{G}^\dagger\right) = \mathfrak{F}^\dagger \mathbb{H} \mathfrak{F}, \tag{51}$$

$$\mathbb{H} = \frac{m_I}{4} \begin{bmatrix} 1+\mu^2 & 1-\mu^2 \\ 1-\mu^2 & 1+\mu^2 \end{bmatrix}, \tag{52}$$

and has discrete spectrum for fixed mass scale

$$\mathrm{Spec}\ \mathbb{H} = \left\{ \frac{m_I}{2}\mu^2, \frac{m_I}{2} \right\}. \tag{53}$$

The internal energy calculated from the Hamiltonian matrix (52) is

$$U = \frac{\mathrm{Tr}(\mathbb{D}\mathbb{H})}{\mathrm{Tr}\mathbb{D}} = \frac{m_I}{4}(\mu^2 + 1). \tag{54}$$

The Hamiltonian matrix $\mathbb{H}$, however, consists constant term $\mathbb{H}_I$

$$\mathbb{H}_I = \frac{m_I}{4}\left[\begin{array}{cc} 1 & 1 \\ 1 & 1 \end{array}\right] \tag{55}$$

which can be eliminated by simple renormalization

$$\mathbb{H} \to \mathbb{H}' = \mathbb{H} - \mathbb{H}_I = \frac{m_I}{4}\left[\begin{array}{cc} \mu^2 & -\mu^2 \\ -\mu^2 & \mu^2 \end{array}\right]. \tag{56}$$

The renormalized Hemiltonian spectrum is

$$\mathrm{Spec}\ \mathbb{H}' = \left\{\frac{m_I}{2}\mu^2, 0\right\}, \tag{57}$$

and straightforward computation of the renormalized internal energy yields the following result

$$U' = \frac{\mathrm{Tr}(\mathbb{D}\mathbb{H}')}{\mathrm{Tr}\mathbb{D}} = \frac{m_I}{4}\mu^2 \equiv U - U_I, \tag{58}$$

where $U_I = \dfrac{m_I}{4}$, which has the Eulerian homogeneity of degree 2

$$U'[\alpha\mu] = \alpha^2 U'[\mu]. \tag{59}$$

In this manner thermodynamics describing space quanta behavior can be formulated in the way typical for the Eulerian systems.

3.2 *Ab initio* thermodynamics of space quanta

Three elementary physical quantities – occupation number N, internal energy U, and entropy S – was just derived, so that one can conclude formal thermodynamics. Actually the entropy (45) is infinite by the presence of formal infinity $\zeta(1)$. Straightforward calculation shows that temperature $T = \delta U/\delta S$ arising from the entropy (45) is dependent on $\zeta(1)$ and initial data mass m_I. Obtained quantity has the finite limit, if and only if we scale initial data mass $m_I \to m_I\zeta(1)$. Because mass m is related to length l by $m \sim 1/l$, the limit $m_I \to \infty$ corresponds with a point object $l_I \to 0$.

However, scaling of initial data is not good physical procedure, *i.e.* has not well-defined physical meaning. It can be shown that the entropy renormalization $S \to -S/\zeta(1)$ in the formal limit $\zeta(1) \to \infty$ gives the equivalent

result for the thermodynamics with no using initial data scaling. The renormalization corresponds to an initial quantum state of an embedding being a point, and yields perfect accordance with the second law of thermodynamics

$$S \longrightarrow S' = \lim_{\zeta(1)\to\infty} \frac{-S}{\zeta(1)} = \frac{1}{2}\left(\frac{\mu^2-1}{\mu^2+1}\right)^2 \geqslant 0. \tag{60}$$

Calculating temperature T of space quanta one obtains

$$T = \frac{\delta U'}{\delta S'} = m_I \frac{(\mu^2+1)^3}{8(\mu^2-1)}, \tag{61}$$

and one sees that initially, *i.e.* for $\mu = 1$, temperature is infinite. It is the Hot Big Bang (HBB) phenomenon. After HBB system is cooled right up until mass scale value $\mu_{PT} = \sqrt{2} \approx 1.414$ and then is warmed. In fact μ_{PT} is the phase transition point, namely, the energetic heat capacity C_U having the form

$$C_U = T\frac{\delta S'}{\delta T} = \frac{\delta U'}{\delta T} = \frac{(\mu^2-1)^2}{(\mu^2-2)(\mu^2+1)^2}, \tag{62}$$

possesses the singularity in the point μ_{PT}. The generalized law of equipartition $\delta U/\delta T = f/2$ establishes degrees of freedom f number

$$f = 2C_U. \tag{63}$$

The Helmholtz free energy $F = U' - TS'$ that is

$$F = -\frac{m_I}{16}(\mu^4 - 4\mu^2 - 1), \tag{64}$$

is finite for finite m_I, increases since $\mu = 1$ till μ_{PT}, and then decreases. So, the thermal equilibrium point is the HBB point $\mu_{eq} = 1$. In the region of mass scales $1 \leqslant \mu < \mu_{PT}$ mechanical isolation is absent, but it is after phase transition $\mu > \mu_{PT}$. Calculating the chemical potential

$$\omega = \frac{\delta F}{\delta N} = -m_I \frac{\mu^3(\mu^2-2)}{2(\mu^2-1)}, \tag{65}$$

one sees that in μ_{eq} it diverges and in μ_{PT} it vanishes. Using of (65) together with the occupation number N and the Helmholtz free energy F yields appropriate free energy defined by the Landau grand potential Ω

$$\Omega = F - \omega N = m_I \frac{3\mu^6 + \mu^4 - 11\mu^2 - 1}{16(\mu^2-1)}, \tag{66}$$

so that the corresponding Massieu–Planck free entropy Ξ can be also derived

$$\Xi = -\frac{\Omega}{T} = -\frac{3\mu^6 + \mu^4 - 11\mu^2 - 1}{2(\mu^2+1)^3}, \tag{67}$$

and consequently the grand partition function Z is established as

$$Z = e^{\Xi} = \exp\left\{-\frac{3\mu^6 + \mu^4 - 11\mu^2 - 1}{2(\mu^2+1)^3}\right\}. \tag{68}$$

The 2nd order Eulerian homogeneity yields the equation of state $PV/T = \ln Z$ and determines the product of pressure P and volume V as

$$PV = -\Omega, \tag{69}$$

so together with the Gibbs–Duhem equation $V\delta P = S'\delta T + N\delta\omega$ allows to establish the pressure

$$|P| = \exp\left\{-\int\left(S + N\frac{\delta\omega}{\delta T}\right)\frac{\delta T}{\Omega}\right\}. \tag{70}$$

Similarly, the first law of thermodynamics, $-\delta\Omega = S'\delta T + P\delta V + N\delta\omega$, and the equation of state (69) determine the volume $|V| = |\Omega|/|P|$, which by positiveness is $V = |V|$. Regarding (69) the pressure $P = |P|$ for $\Omega = -|\Omega| < 0$, and $P = -|P|$ for $\Omega = |\Omega| > 0$, so that

$$P = \begin{cases} \dfrac{m_I^7 a_0}{\mu^2-1}\dfrac{(\mu^2+a_2)^{b_2+1}}{(\mu^2+a_3)^{b_3-1}}|\mu^2-a_1|^{b_1+1} & , \text{ iff } 1 \leqslant \mu \leqslant \sqrt{a_1} \\ \dfrac{-m_I^7 a_0}{\mu^2-1}\dfrac{(\mu^2+a_2)^{b_2+1}}{(\mu^2+a_3)^{b_3-1}}|\mu^2-a_1|^{b_1+1} & , \text{ iff } \sqrt{a_1} \leqslant \mu \leqslant 2+\sqrt{3} \end{cases} \tag{71}$$

where $a_0 \approx 6.676 \cdot 10^6$ and

$$a_1 \approx 1.802\ ,\ a_2 \approx 0.090\ ,\ a_3 \approx 2.046\ , \tag{72}$$

$$b_1 \approx 0.014\ ,\ b_2 \approx 0.410\ ,\ b_3 \approx 1.092\ . \tag{73}$$

For the mass scales $1 \leqslant \mu < \sqrt{a_1}$ P decreases from positive infinity to zero, vanishes in the point $\mu = \sqrt{a_1} \approx 1.343$, and decreases from zero to negative infinity for $\sqrt{a_1} < \mu \leqslant 2+\sqrt{3}$. Regarding the relation $V = |\Omega|/|P|$, V is a fixed parameter and can be established as

$$V = \frac{1}{16 a_0 m_I^6}\frac{1}{|\mu^2-a_1|^{b_1}}\frac{(\mu^2+a_3)^{b_3}}{(\mu^2+a_2)^{b_2}}. \tag{74}$$

Equivalently the thermodynamics of space quanta can be expressed by the size scale $\lambda = \dfrac{1}{\mu}$. There are the relations relating both the scales with an occupation number

$$\lambda = N\left(1 \mp \sqrt{1-\frac{1}{N^2}}\right) \quad , \quad \mu = N\left(1 \pm \sqrt{1-\frac{1}{N^2}}\right), \tag{75}$$

that in the limit of infinite N are equal

$$\lambda_{N=\infty} = \{0, \infty\} \quad , \quad \mu_{N=\infty} = \{\infty, 0\}, \tag{76}$$

so there are two possible asymptotic behaviors. The first case, *i.e.* $\lambda = 0$, $\mu = \infty$, can be interpreted with a black hole as well as with HBB, the second one, *i.e.* $\lambda = \infty$, $\mu = 0$, as stable classical physical object.

4 Discussion

In this paper we have presented the next implication of the simple model of quantum gravity [2]. This algorithm has yielded constructive and plausible phenomenology, that is thermodynamics, in the discussed case describing space quanta behavior. The model applies to all $3 + 1$ splitted general relativistic spacetimes which satisfy the Mach principle, *i.e.* are isotropic. Their importance for elementary particle physics, cosmology and high energy astrophysics is experimentally confirmed; one can say that these are phenomenological spacetimes.

As the example of *ab initio* formulation of thermodynamics we have employed the one-particle approximation of density matrix. The renormalization method was applied for entropy and the Hamiltonian matrix, and has yielded the second order Eulerian homogeneity property. The Landau grand potential Ω and the Massieu–Planck free entropy Ξ was used in the thermodynamic description. Grand partition function Z was established. Thermodynamic volume V was determined as fixed parameter. Other thermodynamical potentials was derived in frames of the entropic formalism, that accords with the first and the second principles of thermodynamics. Physical information following from the thermodynamics of space quanta is the crucial point of the presented construction. Actually the approach of this paper differ from other ones (Cf. *e.g.* [18]) by *ab initio* quantum gravity phenomenology.

In our opinion studying special physical phenomena by the proposed approach seems to be the most important prospective arising from the thermodynamics of space quanta. From experimental point of view the presented considerations possess possible usefulness, because of bosonic systems are common in high energy physics.

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